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Newsvendor Model for Multi-Inputs and –Outputs with Random Yield: Applications to Agricultural Processing Industries

http://www.trc-group.co.th/2017/05/06/thai-jasmine-rice/

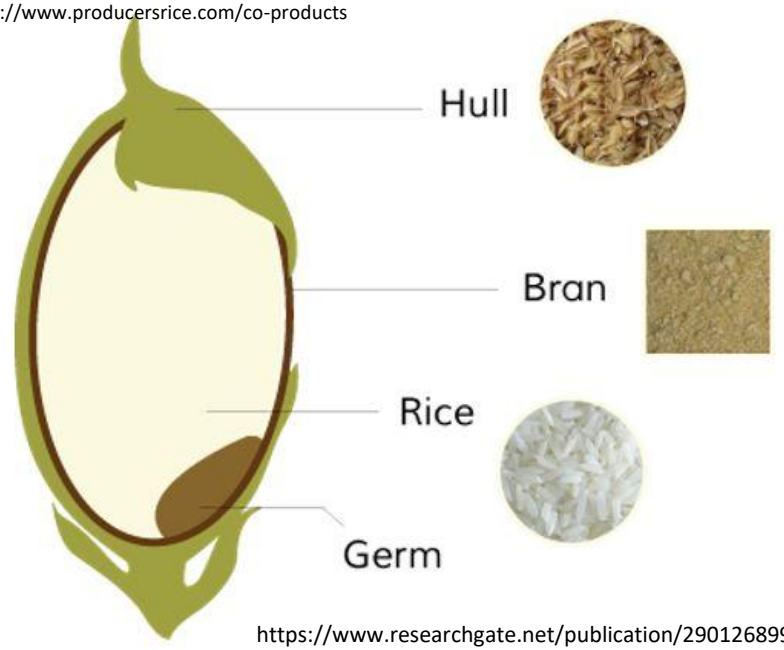
ICORES 2019

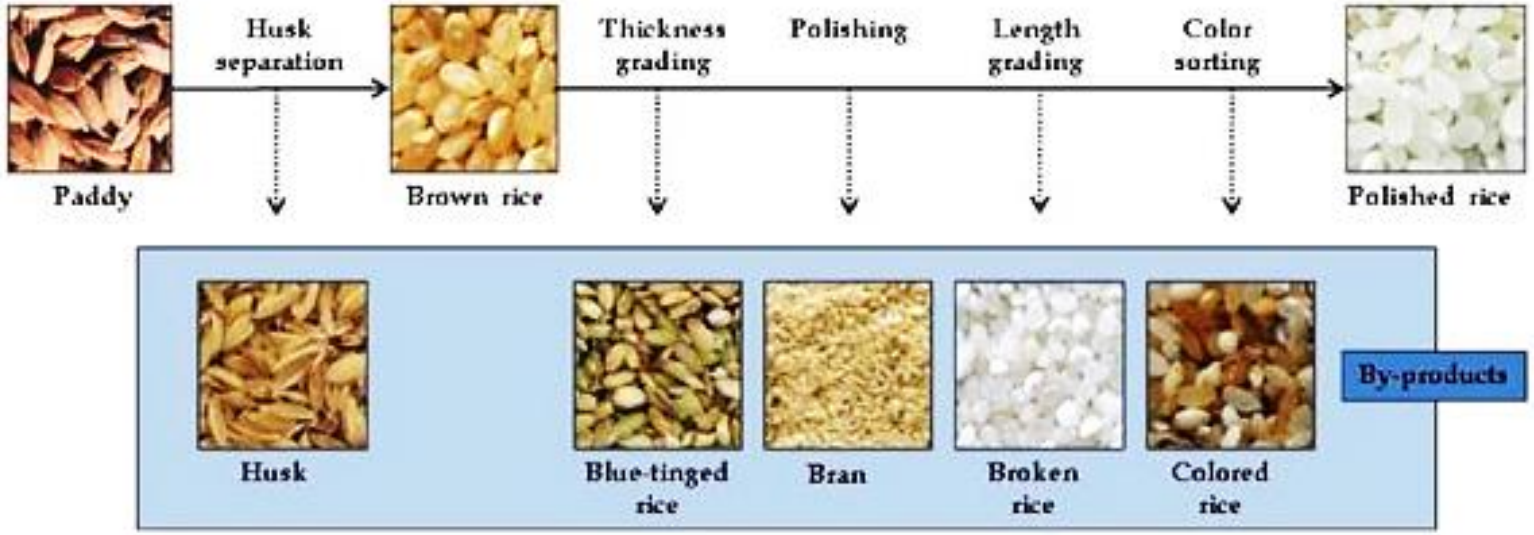
8th International Conference on Operations Research and Enterprise Systems

PRAGUE - Czech Republic 19- 21 February, 2019

Rice processing industry

https://www.producersrice.com/co-products





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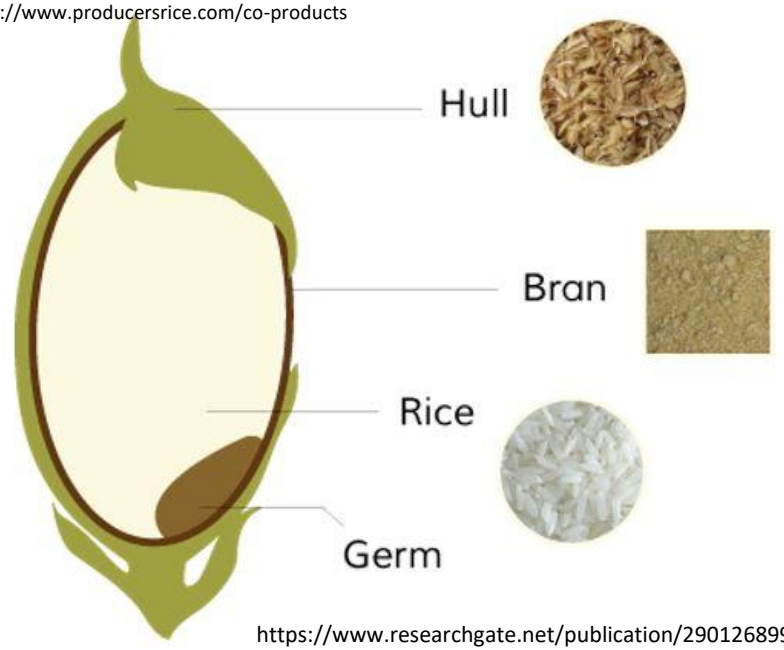

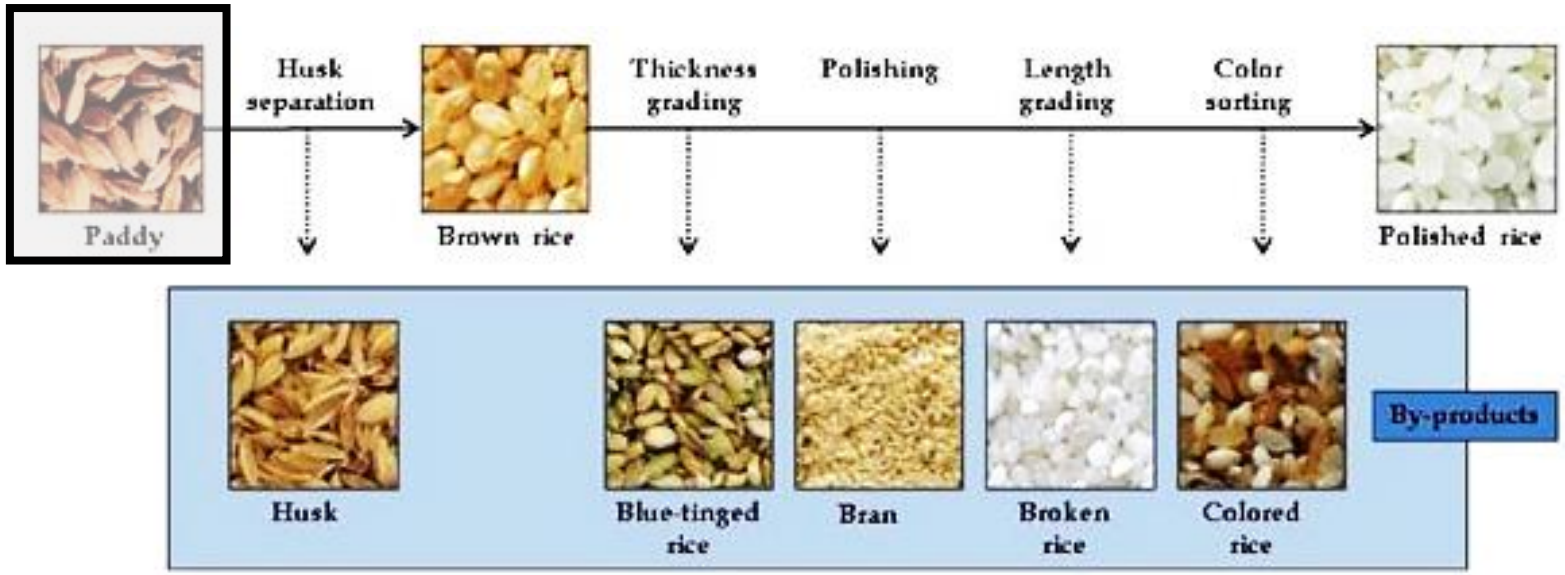

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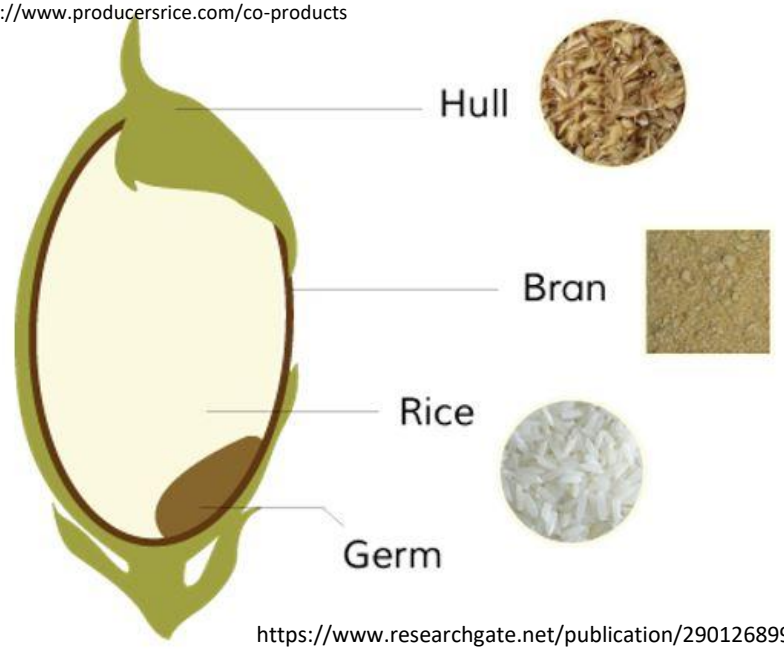

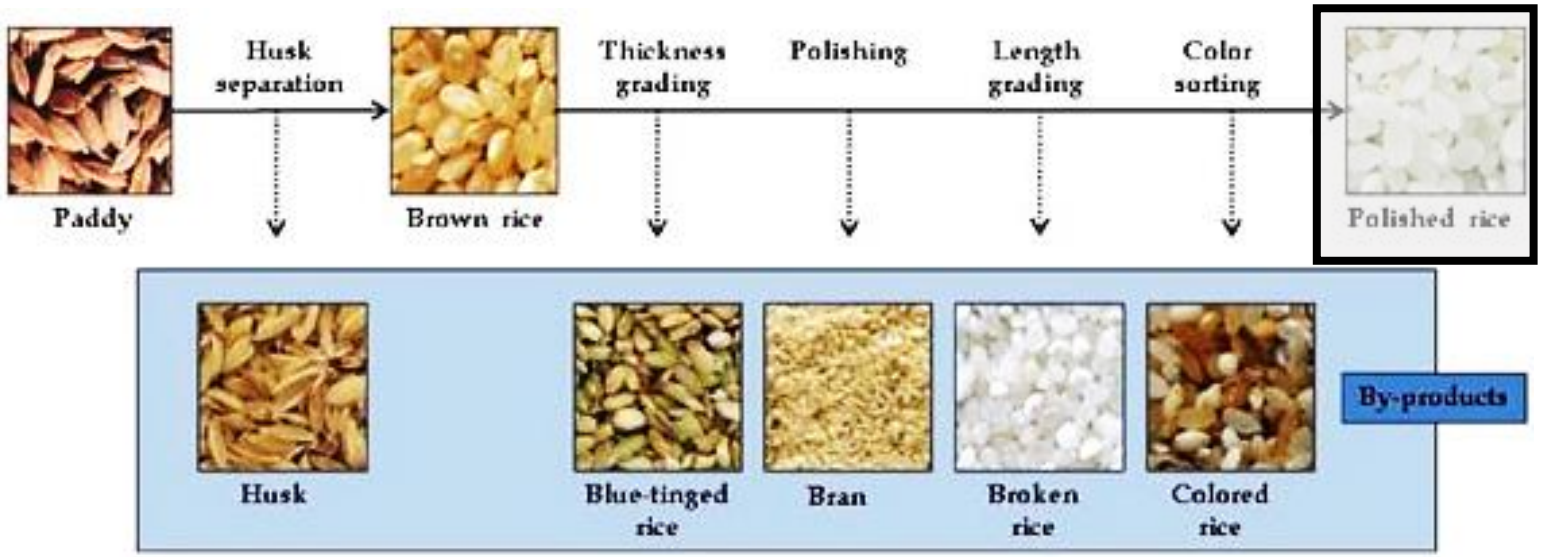

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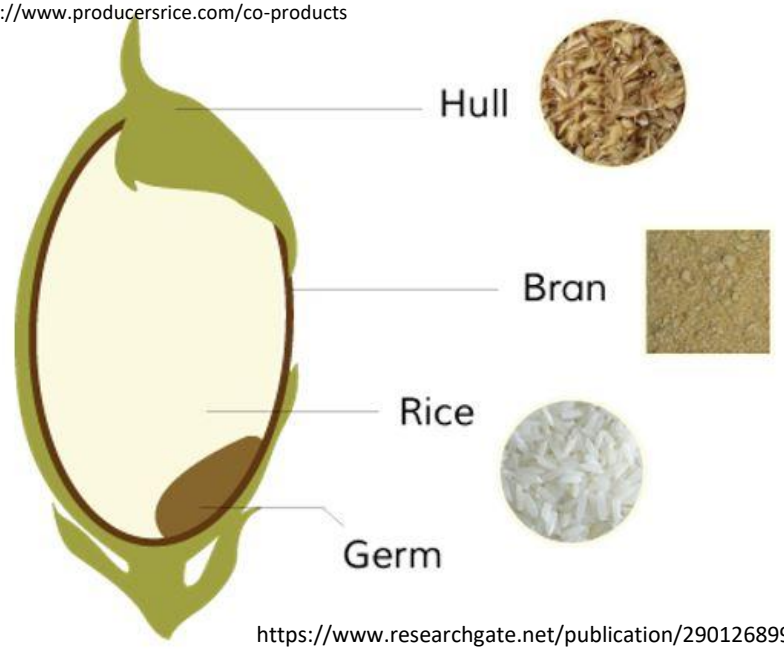

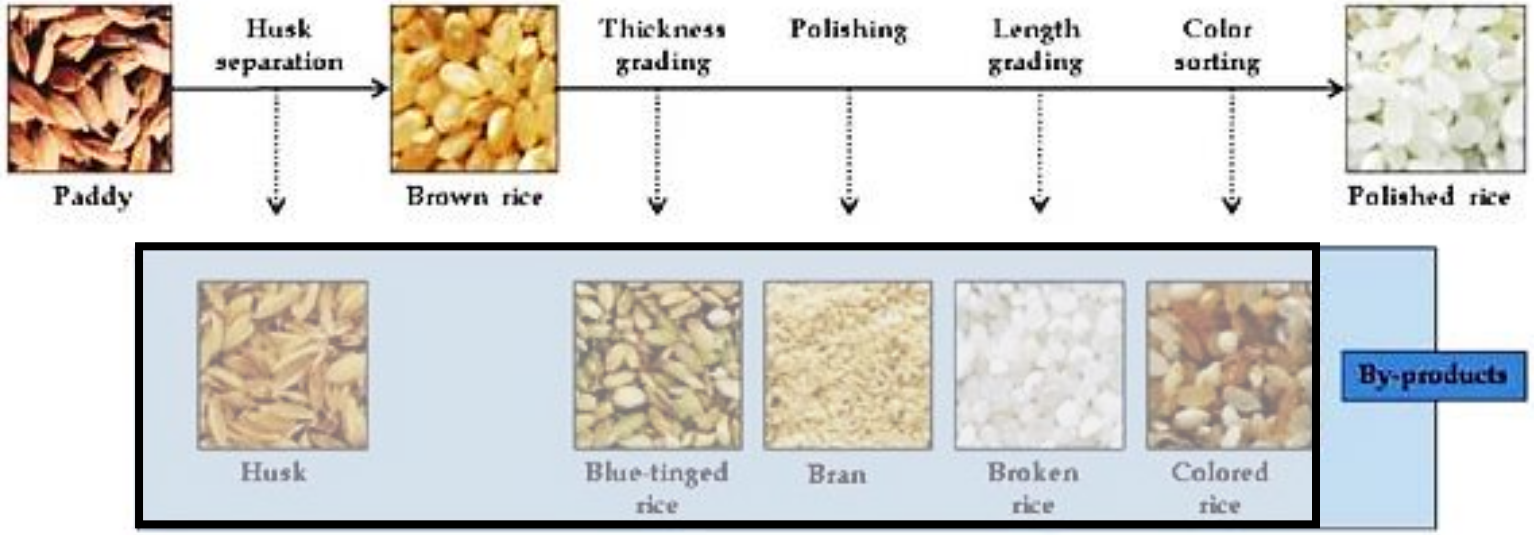

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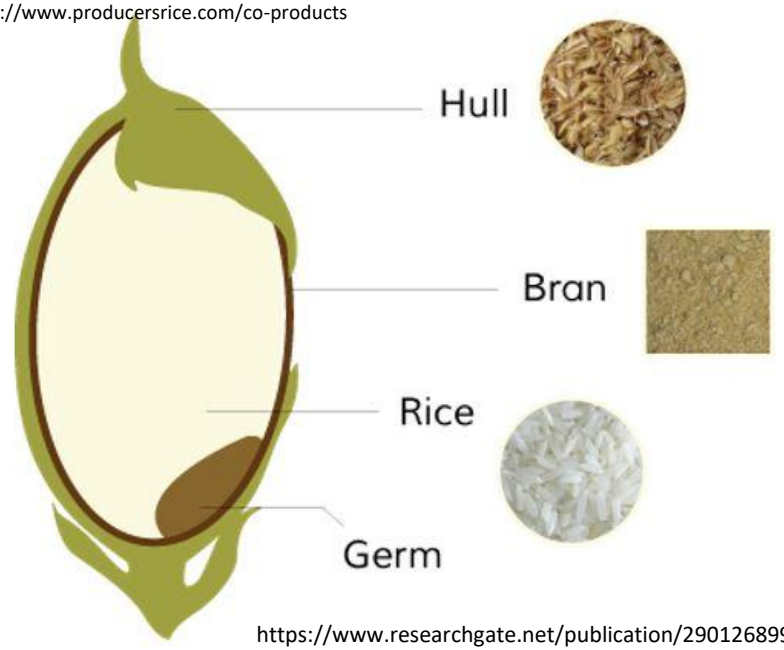

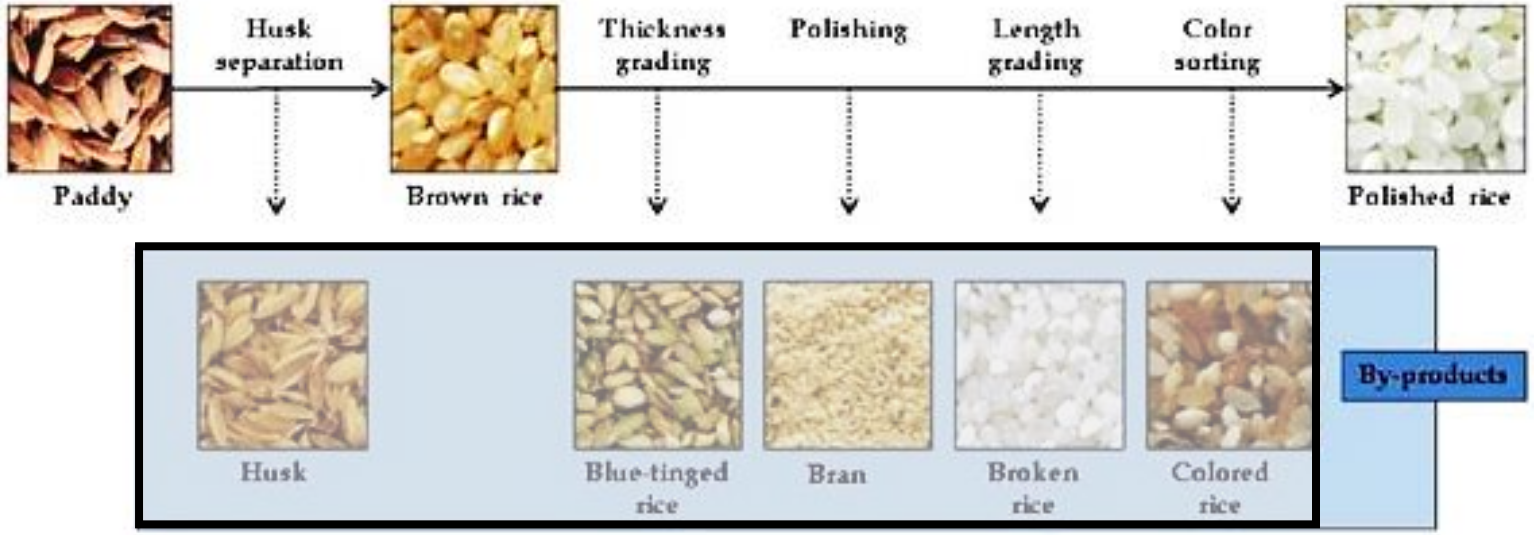

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- The rice bran can be sold to the rice bran oil industry or as animal food for pigs or chickens.
- The rice husk is used in paper production and biomass power generation.

Price of agricultural commodity

KDML105 Paddy price in 2013-2014 at different %MC (THB/ton)

% MC	%Loss\Head Yield	Thai Aromatic rice (KDML105) (gram)										
		50	49	48	47	46	45	44	43	42	41	40
0.0- 15.0	-	21,600.00	21,400.00	21,200.00	21,000.00	20,800.00	20,600.00	20,400.00	20,200.00	20,000.00	19,800.00	19,600.00
15.1-15.4	-	21,600.00	21,400.00	21,200.00	21,000.00	20,800.00	20,600.00	20,400.00	20,200.00	20,000.00	19,800.00	19,600.00
15.5-15.9	7.5	21,438.00	21,239.50	21,041.00	20,842.50	20,644.00	20,445.50	20,247.00	20,048.50	19,850.00	19,651.50	19,453.00
16.0-16.4	15.0	21,276.00	21,079.00	20,882.00	20,685.00	20,488.00	20,291.00	20,094.00	19,897.00	19,700.00	19,503.00	19,306.00
16.5-16.9	22.5	21,114.00	20,918.50	20,723.00	20,527.50	20,332.00	20,136.50	19,941.00	19,745.50	19,550.00	19,354.50	19,159.00
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17.5-17.9	37.5	20,790.00	20,597.50	20,405.00	20,212.50	20,020.00	19,827.50	19,635.00	19,442.50	19,250.00	19,057.50	18,865.00
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19.5-19.9	67.5	20,142.00	19,955.50	19,769.00	19,582.50	19,396.00	19,209.50	19,023.00	18,836.50	18,650.00	18,463.50	18,277.00
35.5-35.9	307.5	13,157.50	13,019.00	12,880.50	12,742.00	12,603.50	12,465.00	12,326.50	12,188.00	12,049.50	11,911.00	11,772.50

Thailand Department of Internal Trade. http://www.dit.go.th/en/AgriCom.aspx

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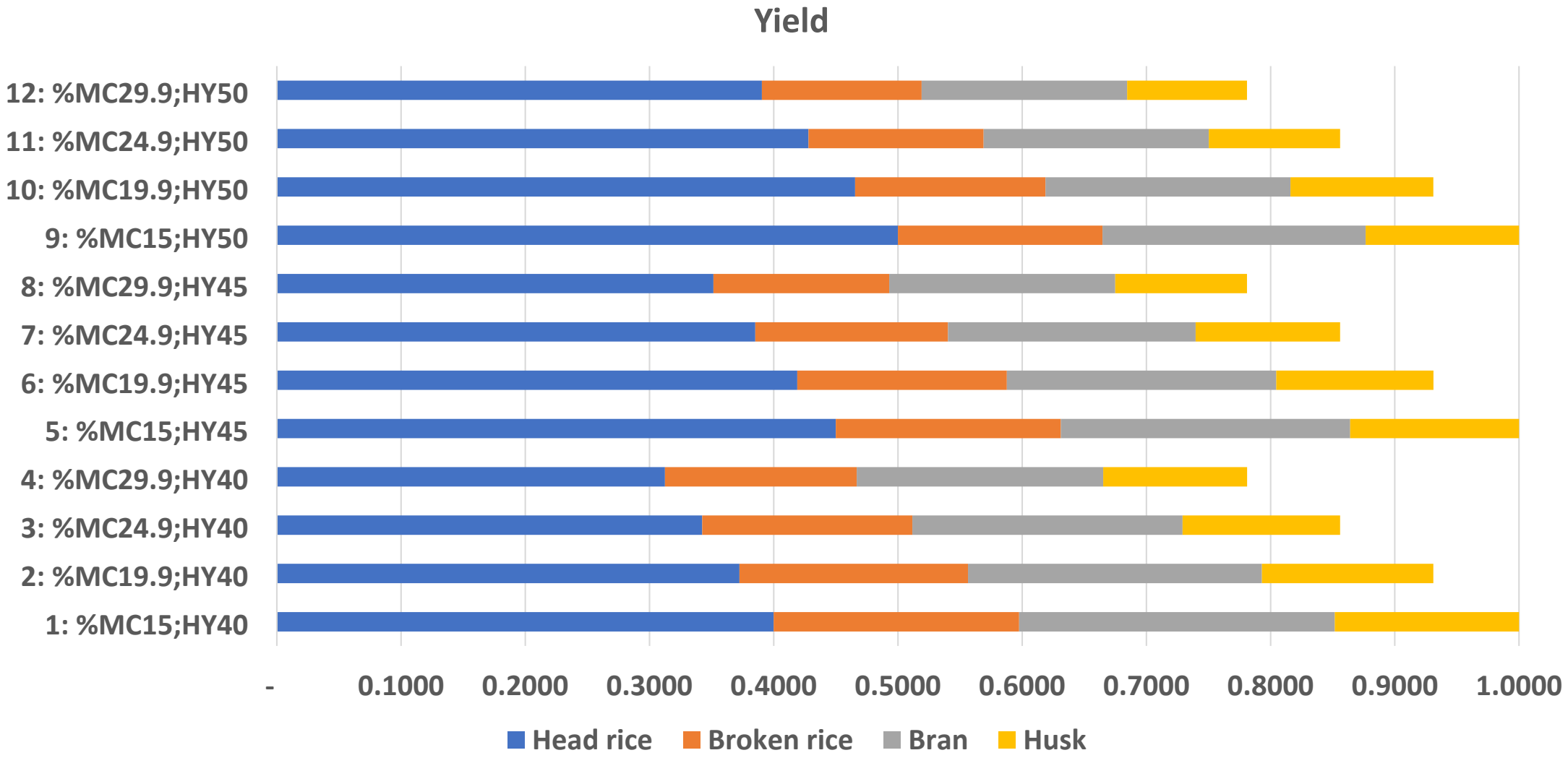
Random yield



Agricultural processing industries

Sugarcane milling

- Yield of raw sugar depends on the cane quality and the juice extraction efficiency.
- Supplier of sugarcane with the greater commercial cane sugar (CCS) receives the higher purchasing price.

Vineyard harvesting

- Grape qualities depend on, e.g., the time of harvest, a wine block, sugar and acidity levels determined by the winery's oenologist.
- Four different types, e.g., *Vins de Prestiges* and *Vins de Charme.*

Fresh fruit

- Different grades depend on sizes.
- Low-quality farm with many aging tress and poor soil quality produces smaller sizes, which are bought at lower prices.

Formulation

- Consider a newsvendor model that produces n outputs from m inputs.
- Let D_j be the random demand of output j for each $j = 1, \dots, n$.
- Let x_i be the purchase quantity of a type-i input for each $i = 1, \dots, m$.
- Determine $x = (x_1, x_2, \dots, x_m)$ before knowing demands $(D_1, D_2, \dots, D_n)$.
- Costs parameters are

c_i = Per-unit cost of a type-i input

p_j = Per-unit selling price of a type-j output

h_j = Per-unit salvage price of a type-j output

g_j = Per-unit lost-sale penalty of a type-j shortage

Formulation

- Costs parameters are

  c_i = Per-unit cost of a type-i input

Table 1: Purchase prices for paddy

%MC	HY 40	HY 45	HY 50	Drying Cost	% Loss
15.0	19600	20600	21600	0.00	0.0
19.9	18277	19210	20142	59.00	6.9
24.9	16807	17665	18522	62.10	14.4
29.9	15337	16120	16902	65.21	21.9

Formulation

- Costs parameters are

p_j = Per-unit selling price of a type-j output

h_j = Per-unit salvage price of a type-j output

g_j = Per-unit lost-sale penalty of a type-j shortage

Table 3: Selling prices, salvage values and penalty costs

Output type (j)	Head 1	Broken 2	Bran 3	Husk 4
p_j	36800	12340	8999	1500
h_j	29872	5000	1000	500
g_j	1000	0	0	0
μ_j	94.00	64.00	53.00	24.00
σ_j	14.10	9.60	7.95	3.60

Formulation: Random yield

- Let $U_{ij}(x_i)$ be the random output of type-j after processing x_i units of a type-i input.
- The expected volume of type-j after processing x_i units of a type-i input is

$$E\left[U_{ij}(x_i)\right] = \int_{\Omega} U_{ij}(x_i, \omega) P(d\ \omega).$$

- The volume of a type-j output given the input $x = (x_1, \ldots, x_m)$ is

$$Y_j(x) = \sum_{i=1}^{m} U_{ij}(x_i)$$

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Formulation: Expected profit

- We assume that the decision maker is risk neutral.
- We want to maximize the expected profit

$$\pi(x) = \sum_{j=1}^{n} E[p_j S_j(x) + h_j W_j(x) - g_j T_j(x)] - \sum_{i=1}^{m} c_i x_i$$

where

$S_j(x) = min(Y_j(x), D_j)$ Type-j sales

$W_j(x) = \left(Y_j(x) - D_j\right)^+$ Type-j leftovers

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Analysis: Convex programming

- Assume that $U_{ij}(x_i) = A_{ij}x_i$.
- Theorem 1. The expected total profit $\pi(x)$ is jointly concave in x.
- The constraint functions are linear, so we have the convex programming.
- There are well-known algorithms to solve the convex programming.

Analysis: Optimality equation

- Define the expected per-unit salvage of the type-i input $h_i' = \sum_{j=1}^{n} h_j E[A_{ij}]$.
- The expected per-unit price p_i' and penalty cost g_i' are defined similarly.
- Define expected *overage* cost $c_i^o = c_i - h_i'$
- Define expected *underage* cost $c_i^u = p_i' - c_i + g_i'$.
- The expected *critical ratio* for type-i input

$$\xi_i = \frac{c_i^u}{c_i^u + c_i^o}$$

- Denote $c_j^s = p_j - h_j + g_j$.
- Theorem 2. An optimal purchase quantity that maximizes $\pi(x)$ is $x_k^* = 0$ for $k \neq i^*$ and $x_{i^*}^* > 0$ which satisfies

$$\int_{\Omega} \left[\sum_{j=1}^{n} c_j^s A_{i^*,j}(\omega) \bar{F}_j(A_{i^*,j}(\omega) x_{i^*}) \right] P(d\,\omega) = c_{i^*}^o$$

where $i^* = \text{argmax}\{\xi_i\}$ the input type with the largest critical ratio.

Sensitivity analysis

- Theorem 3. Assume that the "best" input type remains the same. The optimal order quantities x_{i*}^{*} is larger if one of the following conditions holds (ceteris paribus):

1. The per-unit selling price p_j increases.
2. The per-unit penalty g_j increases.
3. The demand D_j is stochastically large (in the usual stochastic order sense)
4. The per-unit cost c_i decreases.

Conclusion

- We formulate multi-input and –output newsvendor model with random yield.
- We show that the expected total profit is concave and derive the optimal purchase quantities of different types of paddy for a rice mill.
- Extensions
  - Resource constraints (e.g., budget)
  - Mechanism design (in, e.g., contract farming)
  - Risk in agricultural supply chain

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